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Error Exponents in MIMO communications and Matrix Integrals

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Content

Introduction

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Multiantenna Wireless Channels

- The input-output relationship on a multiantenna point-to-point radio channel fed with common transmit signal-to-noise ratio (SNR) , a ected by thermal (AWG) Noise can be written as

$$y = \sqrt{p}\, Hx + z; \qquad (1)$$

- Above, y is the output vector, i.e. the noisy and ltered version of the transmitted input vector x. z represents the AWGN on the receive antenna array, while the generic entry $h_{i \cdot j}$ of the matrix H denotes the channel gain from the j-th transmit antenna to the i-th receive antenna.

Multiple Input Multiple Output (MIMO) System.

Mutual Information

A channel like (1) is characterized by an information flow (measured in bit/s) given by the so-called **mutual information**

$$I() = h(\mathbf{x}) \quad h(\mathbf{x}|\mathbf{y}) = h(\mathbf{y}) \quad h(\mathbf{y}|\mathbf{x}); \tag{2}$$

between the input and the output vectors.

Differential entropy and conditional entropy

In (2),

$$h(\mathbf{x}) = \mathsf{E}_{\mathbf{x}}[\quad \log p(\mathbf{x})]; \tag{3}$$

while

$$h(\mathbf{x}|\mathbf{y}) = \mathsf{E}_{(\mathbf{x};\mathbf{y})}[\quad \log p(\mathbf{x}|\mathbf{y})]. \tag{4}$$

Mutual Information with Gaussian Input and Noise

Coding Theorem

Error-free (i.e. with vanishing error probability) communication can take place at any speed below a characteristic value, depending on the channel structure/statistics, and denoted as **channel capacity** if the

Bounding the Error Probability

The average error probability achievable with maximum likelihood decoding can be bounded as

$$P_e \quad K \exp(\quad n_b n_c E(p; R; n_c)) ;$$

if the realization of H stays xed over n_c channel uses; in this case the input-output relationship over a single code block of length n_b reads as

$$Y = {}^{p^-}HX + Z$$

.

Error Exponent for MIMO

E

Cutoff Rate

$$R_0 = \left. \frac{\log E_c}{n_c} \right|_{r=0; \ =1};$$

Channel Capacity

$$C = @ \ E_c \big|_{r=0; \ =0} = E\left[I\ (\ )\right]$$

Considerations

Error exponent(s) evaluation requires on one hand explicit expressions for the input, the output and the transition densities; on the other hand,

Input Density

$$p_X(X) \propto \exp\left(-\frac{\|X\|^2}{}\right);$$

with uniform power allocation across transmit antennas.

Unitarily invariant channel

For a block-fading channel, fed by a Gaussian input with uniform power allocation, the RCEE related to an observation window of n_b independent fading blocks of n_c channel uses each, can be expressed as

$$E_c \Big/ \int p(\lambda) |I + \lambda|^{n_c} d\lambda ; \qquad (11)$$

where $H^{\dagger}H = U \Lambda U^{\dagger}$.

Let us conclude by making an explicit assumption on Λ.

Giusi Alfano | EE in MIMO

MIMO multiple-clustered scattering
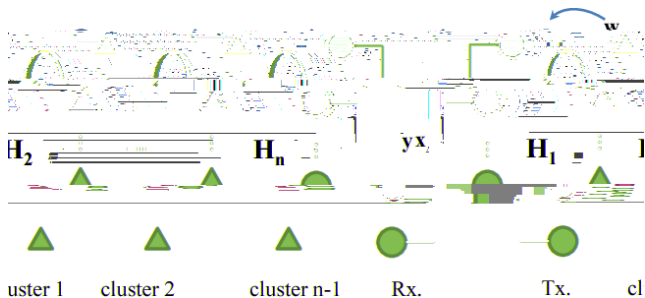
- Objects of noticeable size or groups of objects can be seen as sources of **dominant** contributions in the **multipath fading** process.

- The presence of cluster of dominant scatterers inherently assimilates the channel model, to a cascade of systems.

- The overall impact of multiple-clusters of scatterers is suitably modeled via a product of (independent) Ginibre matrices

$$H = G_n \cdots G_2 G_1 \qquad (12)$$

- Spatial correlation at either ends of the link and the presence of LOS paths makes the model more involved, though it should be taken into account (not reported here, but spectral properties in presence of correlation have been derived by Kuijlaars-Stivigny (2014) and Wei (2015)).

For a block-fading channel with progressive scattering, the RCEE is given by (6) where

$$E \,/\, Z \;\; \frac{(1 \quad pr)(1 + \;)}{p} \qquad :$$

(13)

Open Problems

- Error Exponents expressions depend on coding scheme, and channel state availability;
- If the receiver is not informed, $p(Y|X)$ appears (and it is not Gaussian in general);
- For H non unitarily invariant, the expression is more involved;
- A more general expression of EE corresponding to a **realistic** channel dynamic is to be provided.
- 10.1109/TIT.2018.2803046

Thank you!