

Superbosonization

(a new effective-field method for random matrix and disordered electron systems with local gauge symmetries)

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- Symmetric spaces

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Riemannian symmetric superspaces: definition

Consider sections of $(F_x) \rightarrow M$ (superfunctions),
 M with values $(x) \in (F_x)$

Example

$$1 \quad U \quad / (U \quad U)$$

Example (continued)

V_0	\mathbb{C}^p	0	0	1	1
V_0	\mathbb{C}^q	0	0	1	1
V_1	\mathbb{C}^r	1	1	0	0
V_1	\mathbb{C}^s	1	1	0	0

Lie superalgebra $(\begin{smallmatrix} 0 & 0 \\ 1 & 1 \end{smallmatrix})$ $(\begin{smallmatrix} 1 & 1 \\ 1 & 1 \end{smallmatrix})$
 $(\mathbb{C}^{p \ q} \mid \mathbb{C}^{r \ s})$ $p \ q \mid r \ s$

Important:



$$\left\langle e^{i\text{Tr}HK} \right\rangle_{\text{GOE}} = e^{i\text{Tr}HK} \quad () \quad e^{(b^2/2N) \text{Tr} K^2}$$



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- Background & motivation (continued)

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Pruisken-Schäfer domain

$$Rv = v : \begin{cases} v^t s v = 0 & \text{space-like,} \\ v^t s v < 0 & \text{time-like, } \mathbf{O} \end{cases}$$

Every $O_{p,q}$ diagonalizable matrix R has p space like and q time like eigenvalues.

Encode ordering by motif, e.g., $(R) = \mathbf{O} \infty (p \ q \ 3)$. Associate with each motif a domain D by closure.

Pruisken Schäfer domain $D = \bigcup D$ is a union

of $\binom{p \ q}{p \ q}$ domains. Each D is $O_{p,q}$ invariant.

$D \cap D$ for $\binom{p \ q}{p \ q}$ has co dimension 2.

Statement of result

Let $|dR| = \sum_{i,j} dR_{ij}$ (Lebesgue measure)

Theorem (FWZ). There exists some choice of cutoff function

$$\chi(\cdot) \left(\int \chi(\cdot) \text{sgn}(\cdot) \right) = 1 \quad C,$$

Corollary

Formulation in terms of eigenvalues: Let $R = g g^{-1}$ with $\text{diag}(\lambda_1, \dots, \lambda_{p-q})$ and $g \in \text{SO}_{p,q}$. Volume element

$|dR| = J(\lambda) |d\lambda| |dg|$ where $|d\lambda| = \prod_{i=1}^{p-q} d\lambda_i$ and $|dg|$ Haar

$\text{SO}_{p,q} \cdot J(\lambda) |d\lambda| |dg|$ cs 0 0.35294 0.78431 1.8426 73.431 10.38 11.22 ref

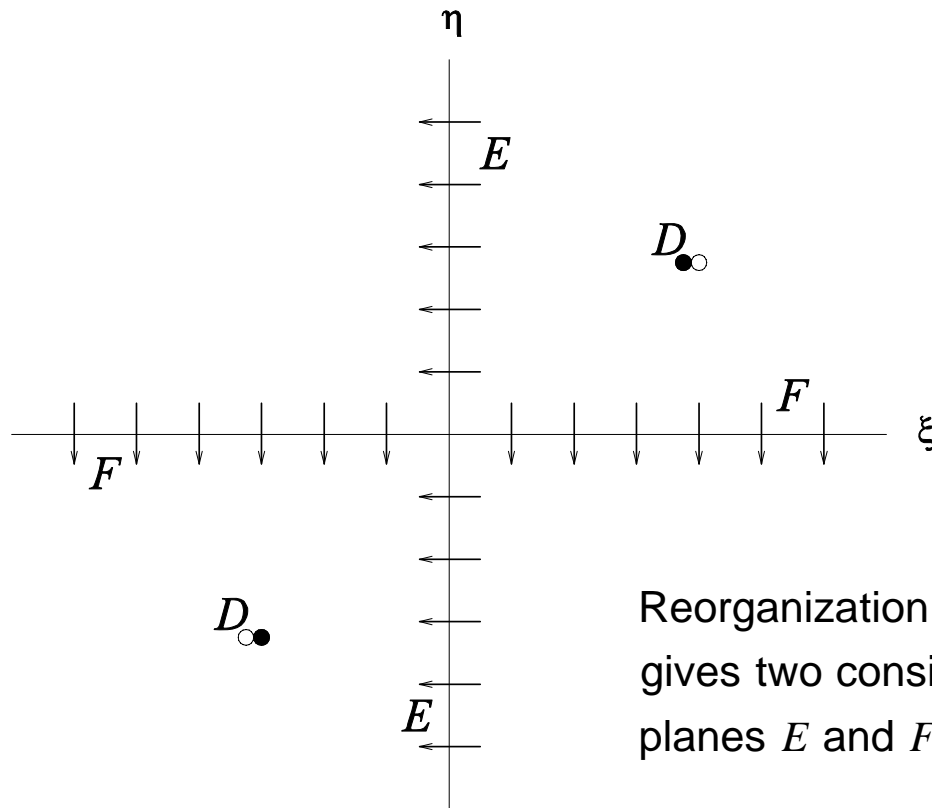


Two domains for $p = q = 1$

$$R \quad \begin{matrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{matrix}$$

Light-cone coordinates:

Reorganization of boundary components



Reorganization of the boundary pieces gives two consistently oriented boundary planes E and F .



Superbosonization

- P. Littelmann, H.-J. Sommers, M.R.Z., Commun. Math. Phys. (in press)
- J.E. Bunder, K.B. Efetov, V.E. Kravtsov, O.M. Yevtushenko, M.R.Z., J. Stat. Phys. 129 (2007) 809

Motivation: supersymmetry method

Central object of the theory is the characteristic function of the

on, $(K) = \int e^{i\text{Tr}HK} d(H)$, where $(Z$ commuting,

anticommuting variables) $K = \int \int d^p Z d^q \tilde{Z}$

Special case: commuting variables only

Let $p = 1, q = 0$ and consider GL_N invariant holomorphic function $f: \mathbb{C}^N \times (\mathbb{C}^N) \rightarrow \mathbb{C}$, $f(z, \tilde{z}) = f(gz, \tilde{z}g^{-1})$, $g \in GL_N$.

Fact (from invariant theory): there exists a holomorphic function $F: \mathbb{C} \times \mathbb{C} \rightarrow \mathbb{C}$ such that $F(\tilde{z} \cdot z) = f(z, \tilde{z})$.

By push forward of the integral one has

$$\int_{\mathbb{C}^N} f(z, \tilde{z}) d^{2N}z = c_N \int_{\mathbb{R}} F(r) r^{N-1} dr \quad (\text{if the integral exists}).$$

generalization: see Fyodorov, Nucl. Phys. B **621** (2002) 643

Special case: Grassmann variables only

Let $F: \mathbb{C} \rightarrow \mathbb{C}$ be a holomorphic function. For a vector $\vec{z} = (z_1, \dots, z_n)$, the Taylor expansion of $F(\vec{z})$ is given by

$$F(\vec{z}) = \sum_{\alpha} \frac{1}{\alpha!} F^{(\alpha)}(\vec{0}) z^\alpha$$

Symmetry argument (heuristic)

Pullback $f: \tilde{Z} \rightarrow Z$. Compare two distributions:

Distribution 1: $\int_{\tilde{Z}} [F] \tilde{z} \, d\tilde{z}$

Distribution 2: $\int_M [F] DQ \, \text{SDet}^{N/2}(Q) F(Q)$

For $g \in \text{OSp}_{2p|2q}$ let $F_g(Q) := F(gQg^T)$.

The transformation behavior is the same:

$$\int_A [F_g] \, \text{SDet}^N(g) \, dA = \int_A [F] \, dA \quad (A = 1, 2).$$



Comments



Application: Wegner's N-orbital model, $U(N)$

Hilbert space $V = \bigoplus_i V_i$, orthogonal projector $P_i : V \rightarrow V_i$
 $\rho(H) = e^{i \text{Tr} HK} d(H) e^{-(1/2N) \sum_{i,j} C_{ij} \text{Tr} K_i K_j}$

reference: