IDENTITIES AND EXPONENTIAL BOUNDS FOR TRANSFER MATRICES

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Luca G- Molinari
Dipartimento di Fisica
Via Celoria Milano

Abstract: Analytic statements can be made on eigenvalues z_i and singular values $_i$ of the transfer matrix T_n of a single general block tridiagonal matrix H:

duality identity and Thouless like identities for $\frac{1}{n}\log|z_i|$ exponents ;

There are constants K, H such that

$$_i > e^{Hn+K}, \quad _{m+i} < e^{-Hn-K} \quad i \quad ...m$$

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Block tridiagonal matrix & its transfer matrix

$$H \quad \begin{bmatrix} A_1 & B_1 & & C_1 \\ C_2 & \ddots & \ddots & \\ & \ddots & \ddots & B_{n-1} \\ B_n & & C_n & A_n \end{bmatrix}_{nm\times nm}$$

$$H \quad E \quad \Rightarrow \quad T_n \; E \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} n+1 \\ n \end{bmatrix}.$$

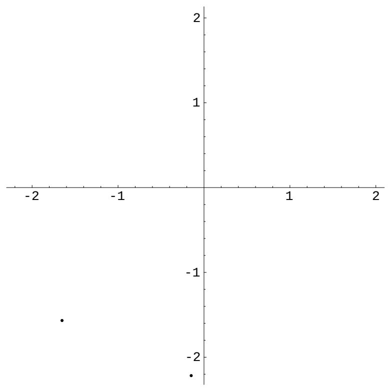
$$n+1 \quad 1, \quad n \quad 0$$

$$T_n \; E \quad \prod_{k=1}^{n} \begin{bmatrix} B_k^{-1} \; E - A_k & -B_k^{-1}C_k^\dagger \\ I_m & \end{bmatrix}_{2m\times 2m}$$

The spectral duality

$$T_{\mathbf{n}} \; E \; \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad z \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \Rightarrow \qquad {}_{\mathbf{n}+1} \quad z \;\; {}_{1}, \qquad {}_{\mathbf{n}} \quad z \;\; {}_{0}$$

Introduce the auxiliary matrix $H \; z$


2
1
-2
-1
1
2
-1
-2

Lemma , Chebyshev

Theorem DMSII is used to give estimates on the singular values of the transfer matrix whose blocks may be represented as blocks of the resolvent of H with corners removed:

transfer matrix resolvent

$$g\ E \quad \begin{bmatrix} E - A_1 & -B_1 & & \\ -C_2 & \ddots & \ddots & \\ & \ddots & \ddots & -B_{n-1} \\ & & -C_n & E - A_n \end{bmatrix}^{-1}$$

$$T_n\ E \quad \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$

$$\begin{bmatrix} -B_n^{-1}\ g_{1,n}\ ^{-1} & -B_n^{-1}\ g_{1,n}\ ^{-1} g_{1,1} C_1 \\ g_{n,n}\ g_{1,n}\ ^{-1} & g_{n,n}\ g_{1,n}\ ^{-1} g_{1,1} C_1 - g_{n,1} C_1 \end{bmatrix}$$

Exponential bounds for singular values $_k$ of T

Lemma Let $t_k\ k$ $...m$ be the singular values of the block T_{11} of $T_n\ E$ then:

$$t_k > \frac{}{K} q^{-n/2}$$

Use the properties interlacing property $_k \geq t_k \geq\ _{m+k}$
T^{-1} is a transes of the blo